Exercise 15

[1] (50 points)

In the shear Alfven wave (k∥B0), the plasma fluid and magnetic field lines move together. Demonstrate this from a single particle motion picture and the time derivative of the magnetic field line equation.

Hints: What is the magnetic field line velocity in the “y” direction? (see figure below). What are the \( E \times B \) drift velocities for electrons and ions? As a reminder, the magnetic field line equation is given by \( \mathbf{d}l \times \mathbf{B} = 0 \), the \( E \times B \) drift velocity can be obtained by time averaging the equation of motion

\[
m_e \frac{d\mathbf{v}}{dt} = q_e \mathbf{E} + \frac{q_e}{c} \mathbf{v} \times \mathbf{B}_0.
\]

[2] (50 points) Linearize the following equations by \( \sim e^{i k \cdot \mathbf{r} - \omega t} \)

\[
\partial_t \rho_M + \nabla \cdot (\rho_M \mathbf{V}) = 0
\]

\[
\rho_M \partial_t \mathbf{V} = -\nabla P + \frac{1}{c} \mathbf{J} \times \mathbf{B}
\]

\[
\nabla \times (\mathbf{V} \times \mathbf{B}) = \partial_t \mathbf{B}
\]

\[
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}
\]

with \( \rho_M = \rho_{M0} \), \( \mathbf{B} = B_0 \hat{z} + B_1 \hat{y} \), \( \mathbf{V} = V_1 \hat{y} \), \( \mathbf{J} = J_1 \hat{x} \), and \( k = k \hat{z} \) to obtain the dispersion relation for the shear Alfven wave.

Compare to the dispersion relation obtained from the two-fluid theory,

\[
\omega^2 = \frac{k^2 V_A^2}{1 + \frac{V_A^2}{c^2}}
\]

and convince that displacement current in Ampere’s law caused the differences. Alfven speed in cgs-Gaussian units is given by \( V_A = B_0/(4\pi \rho_{M0})^{1/2} \).