Exercise 10

[1] (50pts) A long straight conductor carries current $I$. It is in the form a cylinder of radius $R$ with an off-axis cylindrical hole of radius $b$, as shown. The distance between the axis of the cylinder and the axis of the hole is $a$. Find the magnetic field in the hole.

[2] (50pts) A Hamiltonian of a charged particle in (a) non-relativistic case is given by

$$H(p, x) = \frac{1}{2m} \left[ p - \frac{q}{c} A(x) \right]^2 + q \Phi(x)$$

and in (b) relativistic case,

$$H(p, x) = \left[ m^2 c^4 + c^2 \left( p - \frac{q}{c} A \right)^2 \right]^{1/2} + q \Phi(x)$$

where $m$ and $q$ are the mass and charge of the particle, respectively. Confirm that (a) can be resumed from (b). Derive the equation of motion from (a). As a reminder, canonical equation is given by

$$\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i},$$

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i}.$$