[1] (a) Apply Taylor expansion for a function $f(x)$ in the vicinity of $x$ and obtain $f(x + h)$ and $f(x - h)$. Expand them to the order of $h^2$. (b) Using the relations of (a), derive forms for the first and the second order numerical derivatives that are $df/dx$ and $d^2f/dx^2$.

[2] Solve the matrix equation below for $(x, y, z)$ by the Gaussian elimination. First, do a forward elimination (in other words, generate an upper triangle matrix) and second, do a back substitution. You do not need pivoting (change the order of rows) for this matrix.

$$\begin{pmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 10 \\ 19 \end{pmatrix}.$$ 

[3] Normalize the equation of motion

$$m \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B})$$

$$\frac{d\vec{x}}{dt} = \vec{v}.$$ 

Here, $m$ is the mass and $q$ is the charge of the particle. The position and the velocity of the particle are given by $x$ and $v$, respectively. $\vec{B}$ is the magnetic field characterized by the magnetic field strength $B_0$. Employ Cyclotron frequency $\Omega_c = qB_0/m$ and the scale length $a$ for normalization.

After the normalization, the equation should look like

$$\frac{d\bar{v}}{dt} = \bar{v} \times \bar{B}$$

$$\frac{d\bar{x}}{dt} = \bar{v},$$

where bar denotes the normalized values.

[4] (a) Draw a particle trajectory (“by hands”) in the presence of both magnetic field and electric field and demonstrate $E \times B$ drift motion. Hint: for simplicity take $\mathbf{B} = B_z \hat{z}$ and $\mathbf{E} = E_x \hat{x}$ with constant values of $B_z$ and $E_x$. The $E \times B$ drift will be in the $-\hat{y}$ direction.

(b) Draw particle trajectories in the presence of a **non-uniform** magnetic field (magnetic field strength varies spatially) and explain that the electron drift and the ion drift are in the opposite direction ($\nabla B$-drift).

[5] A second order method (modified Euler’s method) in solving ordinary differential equation is given by

$$x(t + h/2) = x(t) + \frac{h}{2} f(t, x)$$

$$x(t + h) = x(t) + hf(t + h/2, x(t + h/2)).$$

Employing Taylor expansions for $x(t + h)$ and $f(t + h/2, x(t + h/2))$, prove that the error of this second order method is given by

$$Error = x(t + h) - x(t) - hf(t + h/2, x(t + h/2)) = \frac{h^3}{24} f'' + O(h^4).$$